

Quantum railroads and directed localization at the juncture of quantum Hall systems

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Abstract

The integer quantum Hall effect (QHE) and one-dimensional Anderson localization (AL) are limiting special cases of a more general phenomenon, directed localization (DL), predicted to occur in disordered one-dimensional wave guides called “quantum railroads” (QRR). Here we explain the surprising results of recent measurements by Kang *et al.* [Nature **403**, 59 (2000)] of electron transfer between edges of two-dimensional electron systems and identify experimental evidence of QRR’s in the general, but until now entirely theoretical, DL regime that unifies the QHE and AL. We propose direct experimental tests of our theory.

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I. INTRODUCTION

It was discovered by von Klitzing, Dorda and Pepper¹ that the Hall conductance of a two-dimensional electron gas (2DEG) in a magnetic field is quantized in integer multiples of the universal quantum e^2/h . This remarkable phenomenon was explained by Laughlin² using a gauge invariance argument. Subsequently, however, Streda, Kucera and MacDonald,³ Jain and Kivelson⁴ and Büttiker⁵ proposed an alternate point of view in which the integer quantum Hall effect (QHE) is explained on the basis of the Landauer theory⁶ of one-dimensional (1D) transport, within the framework of magnetic edge states introduced by Halperin.⁷ These states which derive from the quantized Landau levels of the 2DEG in a strong magnetic field follow the edges of the sample and are the 1D transport channels of these theories. The electrons in these edge channels travel in the *same* direction and therefore cannot be backscattered. Thus as was elucidated by Büttiker,⁵ they are immune from the effects of Anderson localization that normally inhibits propagation of waves or quantum particles through 1D wave-guides in the presence of disorder^{8–10}: They travel along the edge of a disordered sample over macroscopic distances without resistance or dissipation of energy.

Although the QHE and 1D Anderson localization (AL) are mutually antithetical in this fundamental way, it has been shown¹¹ that they are both special cases of a more general phenomenon, namely, *directed* localization (DL). This is a property of disordered 1D wave-guides called “quantum railroads”(QRR) that support *arbitrary* numbers of channels carrying electrons in *opposite* directions.^{11,12} If L channels carry electrons from New York to Los Angeles and M in the opposite direction, it is predicted¹¹ that for $L \leq M$, none of the electrons leaving New York will reach Los Angeles: After multiple scattering events they all return to New York. However of M electrons leaving Los Angeles on average $M - L$ will reach New York while L return to Los Angeles. Thus in such systems the physics of localization acquires a directionality: All electrons traveling in the minority direction behave as if they are localized while some of those traveling in the majority direction are transmitted through the macroscopic disordered system. AL and the QHE correspond to the special cases $M = L$ and $L = 0$, respectively. These important effects have received much attention.^{10,13,14} However, there have been no reports of experimental realizations of the general DL regime ($M > L > 0$) that should unify them.¹⁵ Recent measurements by Kang *et al.*¹⁷ of electron transfer between the edges of two 2DEG’s revealed a richness of unexpected and puzzling phenomena. In this article we use computer simulations to identify the physics behind these observations. We demonstrate that a consistent explanation of the experiment is possible only if the barrier between the 2DEG’s is surrounded by a potential well that supports QRR’s of edge channels exhibiting DL in the general $M > L > 0$ regime. Interplay between DL and electron transfer is reflected directly in the data. Unlike previous theoretical work, the present theory accounts for *all* of the features of the data. It admits simple and direct experimental tests.

In Section II we outline some key puzzles posed by the experiment of Kang *et al.*¹⁷ and present a critique of an important assumption that has been made in previous theoretical attempts to explain the data. In Section III we examine the effects of disorder on electron transfer through the barrier. In Section IV we present self-consistent Hartree calculations of the edge channel energies in the presence of disorder and arrive at a model in which the

barrier between the 2DES's is surrounded by a strong potential well. In Sections V and VI we solve this model in the regime of finite applied bias voltages and show how the physics of directed localization in QRR's in concert with the breakdown of the quantum Hall effect can account for all of the phenomena observed by Kang *et al.*¹⁷ under applied bias. Our concluding remarks are presented in Section VII.

II. PUZZLES POSED BY THE EXPERIMENT

The energies of the 2DEG edge states near the barrier that separates the 2DEG's are shown in Fig.1a for the simplest model¹⁷ of the experimental system in which disorder and interactions between electrons are neglected and no bias is applied between the 2DEG's. Curves with positive and negative slope correspond to edge channels on opposite sides of the barrier, and electrons in these channels travel in opposite directions as shown in the left inset of Fig.1a. Due to conservation of energy and momentum only electrons at crossings of the curves in Fig.1a can transfer through the barrier. At the crossings themselves there are small energy gaps shown in the right inset of Fig.1a. There are no extended states at energies in the gaps. Therefore an electron in a gap cannot travel far along the barrier. Since it also cannot reverse its course it *must* pass through the barrier. Thus for a long, high barrier electrons with energies in the gaps should be transmitted perfectly through the barrier while electrons at other energies should hardly be transmitted at all. As was noted in Ref. 17 the predictions of this simple model disagree with the data: The differential conductance peaks at zero bias (that signal electron transfer through the barrier) persist over ranges of magnetic field far larger than expected from the small widths of the energy gaps. Also the peaks occur at magnetic fields larger than predicted by factors of 2-4. Kang *et al.* conjectured¹⁷ that these discrepancies may be due to disorder, unexpectedly strong spin polarization or a potential well near the barrier.

Pioneering theoretical studies¹⁸⁻²² based on more detailed models have addressed the roles of transport and electron-electron interactions in this system. In these theories,¹⁸⁻²² in order to account for the observed positions¹⁷ of the zero-bias conductance peaks, the 2DEG's were assumed to be *fully* spin-polarized, *at least* in the range of Landau level filling factors ν between 1.1 and 1.5 where the first zero-bias conductance peak was observed. However this assumption is difficult to reconcile with the absence of features due to spin in the data¹⁷ and also with the presence in the data of a prominent feature that will be discussed in Section VI. Physically the assumption means that the *second* 2DEG Landau level must be partly occupied by electrons having one spin orientation before electrons with the opposite spin orientation begin to occupy their *lowest* Landau level. A necessary (*although not sufficient*) condition for this to occur is that the electron spin splitting energy must be larger than the Landau level splitting, i.e., $g\mu_B B/\hbar\omega_c > 1$. But this condition is *not* satisfied by the values of the 2DEG g -factors that have been measured experimentally²³⁻²⁶: The measured values²³⁻²⁶ of the exchange-enhanced²⁷ Landé factor g in 2DEG's in GaAs have ranged up to a maximum value of approximately 6 that corresponds to $g\mu_B B/\hbar\omega_c = 0.2$. Larger g -factors have been reported in quasi-*one*-dimensional quantum wires but the *largest* measured g values even in those systems have not been large enough to be consistent with a fully spin-polarised system with more than one occupied Landau level.²⁸ Thus it seems unlikely that

the 2DEG's of Kang *et al.*¹⁷ were fully spin polarized when more than one Landau level was occupied. Therefore, in this article we explore an alternate possibility that is consistent with the experimentally established properties of the g -factors of 2DEG's that were summarized above.^{23–26} Namely, we assume that the spin splitting in the experiment of Kang *et al.*¹⁷ was considerably smaller than $\hbar\omega_c$ and also smaller than the Landau level broadening (that is due to disorder) so that the spin splitting can be neglected as a first approximation. We will show in Section IV that if this assumption applies then the zero bias data of Kang *et al.*¹⁷ can be explained only if the barrier in their device was strongly charged. However, in that case we are able to explain not only the observations of Kang *et al.* at zero bias,¹⁷ but also all of their results at finite bias¹⁷ including those that have not been accounted for by any previous model.

On the basis of their theoretical analysis Mitra and Girvin¹⁹ have argued that in order to account for the persistence of the experimentally observed¹⁷ zero-bias conductance peaks it is necessary to consider the effects of disorder on electron transfer through the barrier. However, they did not address this issue quantitatively. We shall do this in the next Section.

III. ELECTRON TRANSFER THROUGH THE BARRIER IN THE PRESENCE AND ABSENCE OF DISORDER

Fig.2 shows results of our computer simulations of electron transfer through the barrier obtained using a recursive Green's function technique.³² The solid curve in Fig.2a is the transfer probability T in absence of disorder and e-e interactions. It confirms the paradoxical predictions of the simple model that were outlined at the beginning of Section II: At energies in the gap (shown in the inset, Fig.1a), electrons are transmitted perfectly through the barrier and $T = 1$. Elsewhere T is near zero except for oscillations due to quantum interference near the main peak; see the inset Fig.2a. The period of the oscillations is two orders of magnitude smaller here than in Ref. 18 due to the much longer barrier in our simulations. Fig.2b shows T for the same system but with an additional smooth random potential that models the electron Coulomb interaction with donor ions in the doped layer adjacent to the 2DEG's. Comparing Fig.2b and 2a shows that even a weak, smoothly varying disordered potential alters the electron transfer mechanism dramatically: T now takes the form of a dense array of extremely narrow resonances associated with electron states in the random potential. The dashed curve in Fig.2a shows the combined effect of finite temperature and disorder on the transfer for the same potential as in Fig.2b: The transmission peak is now sufficiently broad to explain the observed persistence of the conductance peaks, and also the absence of features due to spin in the data *if the spin-splitting is small as in our theory*. However it has not shifted significantly from its position in the absence of disorder. Our simulations for other types of disorder and barrier profiles confirmed that the positions of thermally broadened transmission peaks coincide with the energy gaps of the electron edge channels at the barrier, *regardless of the nature of the disorder*. Thus the explanation of the observed locations of the conductance peaks¹⁷ must involve *other* properties of the 2DEG and barrier that control the energies at which the gaps occur.

IV. ELECTRON-ELECTRON INTERACTIONS AND CHARGING AT THE BARRIER

To explore this further we performed self-consistent Hartree calculations of the edge channel energies, treating the effects of disorder in a mean-field approximation: We assumed that disorder broadens the density of states of each Landau level into a Gaussian distribution whose center tracks the position-dependence of the Hartree potential.³³ This resulted in a downward shift of the edge channel energies and gaps relative to their positions in Fig.1a due to the electron density in the barrier being depleted relative to the 2DEG outside the barrier and this depletion resulting in an electrostatic well around the barrier. However, the size of the shift was insufficient to explain the observed¹⁷ positions of the zero bias conductance peaks.

To characterize the size of the discrepancy physically, we repeated our calculation assuming an *additional* positive charge density ρ to be present inside the barrier such as might be introduced by doping the barrier in the plane where it was cleaved during fabrication of the sample. This yielded a downward shift of the energy gaps sufficient to explain the observed positions¹⁷ of the conductance peaks (for *spin-unpolarized* 2DEG's) for $\rho = 11 \times 10^{11} e \text{ cm}^{-2}$. This value of ρ is so much ($5.5\times$) larger than the charge density of the 2DEG that it is unreasonable to expect *any* electronic many-body effect (i.e., correction to Hartree theory) to result in an effective potential well (whether spin-dependent or not) that could mimic the electrostatic well associated with this large positive charge that is required to account for the experimental data. We believe that this rules out *all* potential explanations of the observed positions of the conductance peaks that rely primarily on many-body corrections, including any explanation that involves strongly spin-polarized electron systems. Since the sample was made using a new, still incompletely understood process we consider the possibility of a *materials-related* potential well that we have modeled by the above charge density ρ to be reasonable. This potential well is an important part of our model of the device of Kang *et al.*¹⁷ that we solve below. (Recall that previous theories^{18–22} relied instead on the assumption that the 2DEG's are fully spin polarized in a range of values of $\nu > 1$ to explain the observed positions¹⁷ of the conductance peaks at zero bias. That assumption, as was explained in Section II, is not consistent with the experimental literature on spin polarization in 2DEG's and has other drawbacks.) We note that the possibility that some inadvertent doping may have occurred at the cleaved interface is consistent with the measured mobility ($10^5 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$) of the 2DEG's¹⁷ that is unexpectedly low for a state of the art GaAs device today. The disordered potential associated with such doping would contribute to the Landau level broadening discussed in Section III to which we and others¹⁹ have attributed the experimentally observed¹⁷ persistence of the differential conductance peaks at zero bias. Furthermore disorder near the barrier plays a key role in the physics of directed localization¹¹ that as we show in Section V is able to account for the fact¹⁷ that tunneling between some pairs of Landau levels is observed experimentally while for other pairs it is not observed. This is an important feature of the data¹⁷ that to date has not been explained in any other way.

V. ELECTRONIC STRUCTURE AND THE PHYSICS OF DIRECTED LOCALIZATION AT FINITE APPLIED BIAS

The theories proposed to date^{18–22} have either not treated the case where a finite bias voltage was applied across the barrier in the experiment of Kang *et al.*¹⁷ at all or have yielded qualitative inconsistencies with the data¹⁷ in that regime. Here we shall apply the model with the charged barrier proposed in Section IV to the case of finite applied bias voltages. We shall show that this model is able to account for all of the features of the data.¹⁷

Examples of our self-consistently calculated edge channel Hartree energies under applied bias are shown in Fig.1b,c. The bias V results in a difference eV between the Fermi levels $E(S)$ and $E(D)$ on the source and drain side of the barrier. The energy of each edge channel now exhibits a *minimum* due to the potential well near the barrier. Since the electron velocity is $v = dE/d(\hbar k)$ this means that edge channels can carry electrons in *opposite* directions along the *same* side of the barrier. Thus QRR's in the *general* ($M > L > 0$) DL regime are realized. Two examples are the QRR's near $E(S)$ on the source side in Fig.1b and c. In both cases there are 2 forward moving channels (the $N = 0$ and 1 channels at w, x, u and f) and 1 backward channel (the $N = 1$ channels at z and b). Thus for an impenetrable barrier, DL theory would predict¹¹ half of the electrons entering the QRR in the forward direction to be reflected and half to be transmitted through the QRR. However, at edge channel crossings transfer through the barrier competes with the forward and back-scattering alternatives of DL. As will be explained below, this competition results in *selective* suppression of the transfer at certain edge channel crossings, in agreement with the conductance data.¹⁷ For example, transfer is suppressed at crossing x (denoted 1,0 in Fig.3) but not at u (0,1 in Fig.3). This difference can be understood physically in terms of the unitarity of the scattering matrix that underpins the predictions of DL theory: Electrons enter the QRR in the two forward channels. If the barrier is impenetrable, then after multiple scattering events these electrons fill a single effective forward channel and a backward channel. However the $N = 1$ forward edge states near f and x and the backward edge states near b and z have low velocities and are thus strongly mixed and localized by disorder. Therefore the effective forward channel of DL theory consists, for the most part, of the $N = 0$ edge state at u or w . Now consider a barrier that is *not* impenetrable. Then it is crucial whether an energy gap opens in the $N = 0$ edge channel at $E(S)$, blocking propagation through the forward DL channel. This happens in Fig.1c (the gap is at u) so that electrons in the forward DL channel are blocked and must transfer through the barrier. But in Fig.1b the gap is at x so the forward DL channel is still open (at w) and forward propagation dominates over transfer through the barrier which is only weakly transmitting. Thus transfer at crossing x is suppressed. The absence of the associated conductance peak in the data of Kang *et al.*¹⁷ is evidence of a QRR in the general ($M > L > 0$) DL regime: It is inconsistent with models in which all edge states on the same side of the barrier travel in the same direction because for such models *unitarity* requires that transfer through the barrier *not* be suppressed at *any* edge channel crossing. This is why the absence of this peak in the data¹⁷ could not be explained by previous theoretical work.¹⁸

Backscattering also occurs at the crossing of the two $N = 0$ channels in Figs.1b and c. However this crossing is well below the source 2DEG Fermi level. Thus, if the bias is somewhat larger than in Fig.1b so that the crossing is above $E(D)$, electrons from occupied

states at higher energies *along the entire length of the source side of the barrier* can decay to states at this crossing and then transfer through the barrier. Thus we do not expect the conductance peak associated with this crossing to be suppressed by localization effects.

The locations of the conductance maxima predicted by our calculations of the edge channel crossings are compared with the observed positions¹⁷ of the conductance peaks in Fig.3. Blue (red) curves indicate edge channel crossings at which transfer is (not) expected to be suppressed according to the above considerations. There is an obvious one-to-one correspondence between the red curves and the loci of experimental conductance maxima and good quantitative agreement at Landau level fillings >1.14 for positive and small negative bias. However the red curves do not follow the bell-shaped structure at lower Landau level fillings or exhibit the asymmetry seen experimentally between positive and negative bias. In Section VI we explain these deviations as manifestations of the breakdown of the quantum Hall effect (QHE).

VI. BREAKDOWN OF THE QUANTUM HALL EFFECT

The bias voltage in our self-consistent calculations is evaluated between points in the vicinity of the barrier. In the quantum Hall regime the 2DEG's are perfect conductors. Therefore the measured bias is equal to the bias voltage in our theory. When the QHE breaks down, the 2DEG's become resistive and the measured bias acquires a contribution due to potential differences within the 2DEG's *in addition* to the potential drop across the barrier; thus the magnitude of the experimentally measured bias voltage should exceed the theoretical value. If the magnetic spin-splitting is *small* (as in our theory), the QHE breaks down at zero bias when the Landau level filling falls below a value somewhat larger than 1 that depends on the 2DEG mobility.^{13,14} Thus we explain the bell-shaped structure in the experimental data below the Landau level filling of 1.14 where the magnitude of the experimental bias voltage at the conductance peak abruptly begins to exceed the theoretical one as a manifestation of the breakdown of the QHE. We note that no explanation of the bell-shaped structure was offered by previous theories. The present explanation is not compatible with their assumption that the 2DEG is fully spin-polarized since in that case the QHE would break down at a quite different value of the Landau level filling. The value of the Landau level filling at which the QHE breaks down can be determined independently by resistance measurements of the 2DEG^{13,14}; our explanation of the bell-shaped feature can be tested directly in this way. This is an *important* experimental test of the assumptions underlying *all* of the theories that have been proposed to date since it should be able to determine whether the system is fully spin polarised as has been assumed previously^{18–22} or weakly spin polarised as we have suggested here. We also predict similar behavior at Landau level fillings below 3 (outside the range of the data in Ref. 17) where the QHE should also break down. That the experimental features at higher bias are asymmetric and occur at somewhat larger bias values than the theoretical ones can also be understood within our framework since the QHE breaks down eventually with increasing bias at *all* Landau level fillings and does so in a *sample-dependent* way.³⁵

VII. CONCLUSIONS

In conclusion, we have presented a theory that offers a resolution of *all* of the puzzles posed by recent measurements of electron transfer between the edges of two-dimensional electron systems and have identified the first experimental evidence of the general directed localization regime that unifies the integer quantum Hall effect and one-dimensional Anderson localization. We have also proposed a simple measurement that should be able to settle unambiguously the important issue of the role that spin plays in this system.

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- ³³ It is well established³⁴ that for 2DEG's with mobility values such as that estimated for their device by Kang *et al.*,¹⁷ disorder gives rise to a broadening of the Landau levels whose magnitude is a significant fraction of $\hbar\omega_c$ although the density of states is not vanishingly small in any energy range between the Landau levels. In the present self-consistent calculations this effect of disorder was modeled as a Gaussian broadening of the Landau level density of states with a half-width arbitrarily chosen to be $0.39\hbar\omega_c^*$ (ω_c^* is the value of ω_c at $\nu = 2.3$). However, we carried out some calculations for other values of the half-width as well and found our results not to be sensitive to this parameter.
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FIGURES

FIG. 1. Landau levels $N = 0, 1, 2, \dots$ become edge channels near barrier. a, Edge state energies E at zero bias vs. electron wave vector k along barrier in the simplest model of 2DEG's and barrier. $l_B = (\hbar/eB)^{1/2}$. Left inset: 2DEG's, barrier and edge channels. Right inset: Energy gap at edge channel crossing. b and c, Edge channel energies for biases V between the 2DEG's. Dashed lines are source and drain Fermi energies.

FIG. 2. Electron transfer probability through barrier 91\AA thick, 221 meV high¹⁹ and 18.2 microns long at $B = 6\text{Tesla}$ vs. energy. Solid curve in a (b) is for no disorder (a random potential W with correlation length 274\AA and $|W| < \hbar\omega_c$) at 0K . Inset: Expanded view of narrow peak in a. Dashed curve in a is transfer probability at 0.5K for same potential as in b; the broad peak shows weak residual mesoscopic structure.

FIG. 3. Comparison of theory (colored curves) and maxima (black ellipses) of measured conductance¹⁷ for 2DEG's with density $2 \times 10^{11}\text{cm}^{-2}$. i,j indicate transfer from (to) an edge channel derived from Landau level i(j).

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